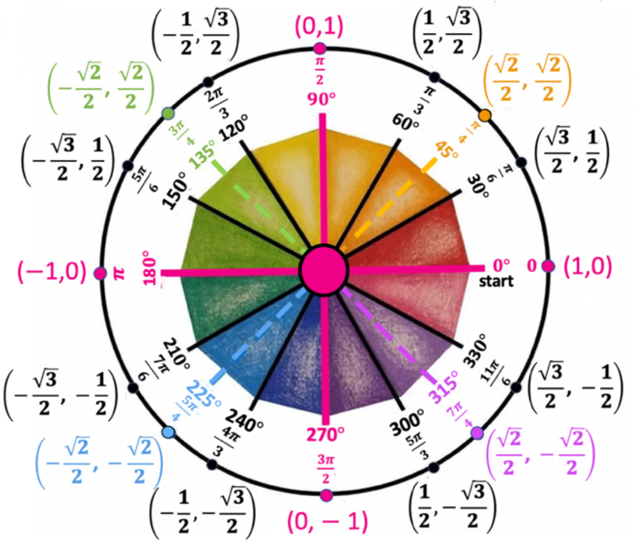
Unit Circle: $(\cos x, \sin x)$



How do we use the unit circle?

Locate the **start** place at 0°. This is our starting point.

The coordinates indicated at each of the angles give us the value of the trig functions (cos and sin) for that particular angle. The angles have been given in degrees and radians (the corresponding radian measures are right above the ° measures and have just been included for completeness - most courses will use degrees to introduce this topic so you can ignore these parts at first).

The x coordinate gives us the value of cos and the y coordinate gives us the value of sin for each angle shown. We go round in an anti-clockwise direction to get the values of sin and cos for angles from 0° to 360° as shown.

Note:

- We can also go around the unit circle in a clockwise direction will find the trig values for negative angles such as -30, -60, $-90°$ etc. $-30°$ clockwise will give the same value as 330° anti-clockwise.
- We can also go around the unit circle again to find the trig values for angles bigger than 360° such as 390°, 405°, 420°, 450° etc.

Summary Table Of Values For The Unit Circle Above

Multiples of 30° and 45°: $30°, 45°, 60°$ etc

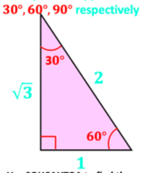
	30°	45°	60°	120°	135°	150°	210°	225°	240°	300°	315°	330°
$\sin\theta$ (y coordinate)	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$
$\cos\theta$ (x coordinate)	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\theta$ $\left(=\frac{\sin\theta}{\cos\theta}\right)$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$

Multiples of 90°: $90°, 180°, 270°, 360°$ etc

	0°	90°	180°	270°	360°
$\sin\theta$ (y coordinate)	0	1	0	-1	0
$\cos\theta$ (x coordinate)	1	0	-1	0	1
$\tan\theta$ $\left(=\frac{\sin\theta}{\cos\theta}\right)$	0	∞	0	∞	0

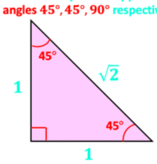
Alternative method for finding angles of $30°, 45°$ and $60°$ etc : Use SOHCAHTOA

Angles of 30°, 60°, 90°:
Memorise the side lengths 1, $\sqrt{3}$, 2. These correspond to opposite the angles 30°, 60°, 90° respectively



Use SOHCAHTOA to find the values for $\sin 30°, \cos 30°, \tan 30°$ $\sin 60°, \cos 60°, \tan 60$

Angles of 45°, 45°, 90°:
Memorise the side lengths 1, 1, $\sqrt{2}$. These correspond to opposite the angles 45°, 45°, 90° respectively



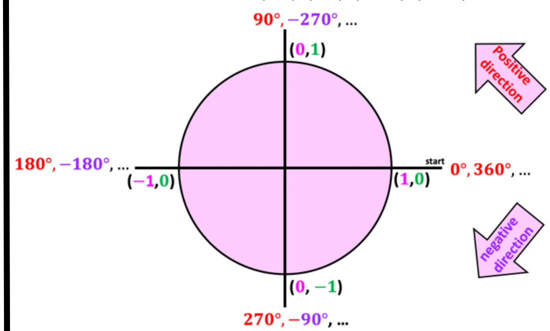
Use SOHCAHTOA to find the values for $\sin 45°, \cos 45°, \tan 45°$

Key:
Use SOHCAHTOA to find the values for the angles $30°, 45°, 60°$

$$\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}, \tan\theta = \frac{opp}{adj}$$

Alternative method for multiples of 90° :

Plot the 4 coordinates (0,1), (1,0), (−1,0), (0,−1)



Key:

$(x, y) \Rightarrow (\cos\theta, \sin\theta)$

This means the x coordinate gives us the value of cos
the y coordinate gives us the value of sin
for any of the angles θ (the angles are represented on the axes)

For example

$\cos 90° = x$ coordinate at $90° = 0$

$\cos 180° = x$ **coordinate** at $180° = -1$

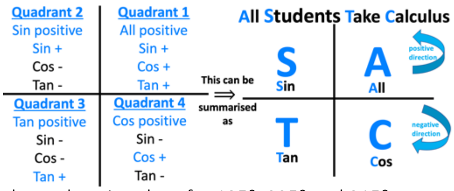
$\cos 270° = y$ coordinate at $270° = -1$

We can also find negative angles by going clockwise

$\cos(-180)° = y$ coordinate at $-180° = 0$

Alternative Method For Larger Angles (over 90°)

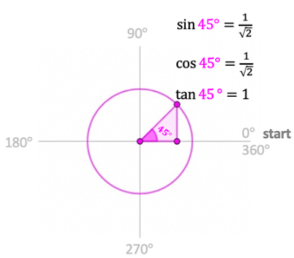
From knowing the values of the trig functions for the acute angles above (30°, 45° and 60°), we can find the values of the trig functions for larger angles. First, recall the signs of the trig functions in each quadrant shown on the right. Also recall that going anti-clockwise gives us positive angles and clockwise gives us negative angles.



For example, if we know the trig values for 45°, we can also work out the trig values for 135°, 225° and 315°.

45°

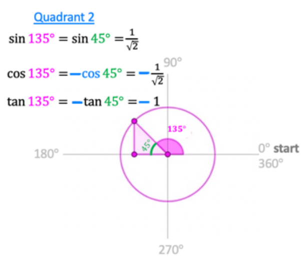
We know these values since we are meant to memorise the values of trig functions for angles 30, 45°, 60° and 90°. We draw an angle of 45°.



135°

We draw an angle of 135° which leaves an angle of 45° in the 2nd quadrant. We know that the trig values will be the exact same as for 45°, we just have the worry about the signs in the 2nd quadrant.

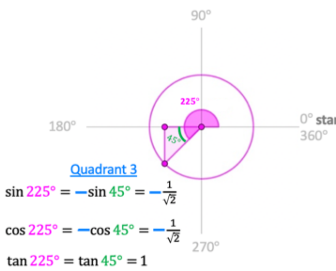
sin is positive, but cos and tan are negative in quadrant 2



225°

We draw an angle of 135° which leaves an angle of 45° in the 3rd quadrant. We know that the trig values will be the exact same as for 45°, we just have the worry about the signs in the 3rd quadrant.

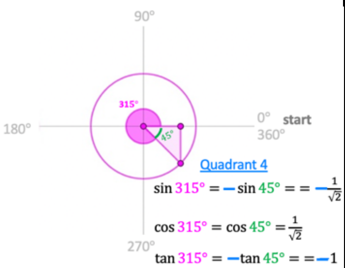
tan is positive, but sin and cos are negative in quadrant 3



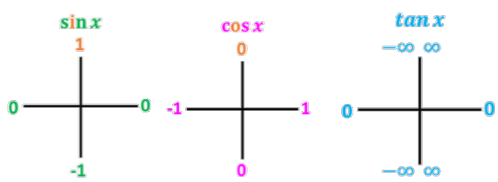
315°

We draw an angle of 315° which leaves an angle of 45° in the 4th quadrant. We know the trig values will be the same as for 45°, we just have the worry about the signs in the 4th quadrant.

cos is positive, but sin and tan are negative in quadrant 4



These values can be summarised in 3 simple diagrams



$\sin(-90°) = -1$	$\cos(-90°) = 0$	$\tan(-90°)$ =undef
$\sin 0° = 0$	$\cos 0° = 1$	$\tan 0° = 0$
$\sin 90° = 1$	$\cos 90° = 0$	$\tan 90°$ =undef
$\sin 180° = 0$	$\cos 180° = -1$	$\tan 180° = 0$
$\sin 270° = -1$	$\cos 270° = 0$	$\tan 270°$ =undef
$\sin 360° = 0$	$\cos 360° = 1$	$\tan 360° = 0$
etc	etc	etc

We can do the same thing with 30° to get the values for the angles 150°, 210°, 330° etc (just like we did for 45° on the page above on the left to get the values for the angles 135°, 225° and 315°). We can also do the same thing with 60° to get the values for the angles 120°, 240°, 300° etc . The unit circle always works for negative angles, just go clockwise!

Worried and wondering how you can remember all this? We only need to remember the trig values for $0°, 30, 45°, 60°$ and $90°$.

Pattern to help remember the table on the right					
	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\tan\theta$	$\frac{0}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{\sqrt{3}}$	$\frac{\sqrt{3}}{1}$	$\frac{\sqrt{3}}{0}$

simplify the values

$\Longrightarrow$

Memorise this table					
	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Take note of the pattern in the table above:

- For $\cos\theta$,we just reverse the order of $\sin\theta$
- $\sin\theta$ and $\cos\theta$ always have a square root in numerator and the denominator is always 2.
  0,1 2, 3, 4 in the numerator for sin goes to 4,3,2,1,0 for cos

$$\begin{array}{c|ccccc} & 0° & 30° & 45° & 60° & 90° \\ \hline \sin & 0 & 1 & 2 & 3 & 4 \\ \cos & 4 & 3 & 2 & 1 & 0 \\ \hline & & & 2 & & \end{array}$$

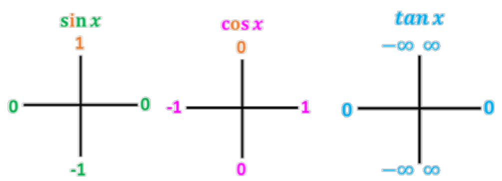
- To help remember $\tan\theta$, you can also just do $\frac{\sin\theta}{\cos\theta}$
  The colour pattern can also help you to remember $\tan\theta$.

We can use the symmetry of the unit circle as explained on the previous page, to get the trig values for angles larger than 90° and also negative angles.

Graphs of $\sin x, \cos x, \tan x$:

You'll also need to know the graphs of trig functions. Knowing the trig values for angles which are multiples of 90° helps to easily remember the graph. On a trig graph, the x axis represents the angles and the y axis represents the trig values.

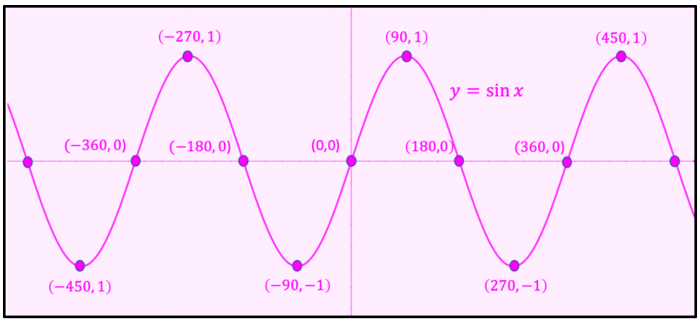
Recall, that these values can be summarised in 3 diagrams:



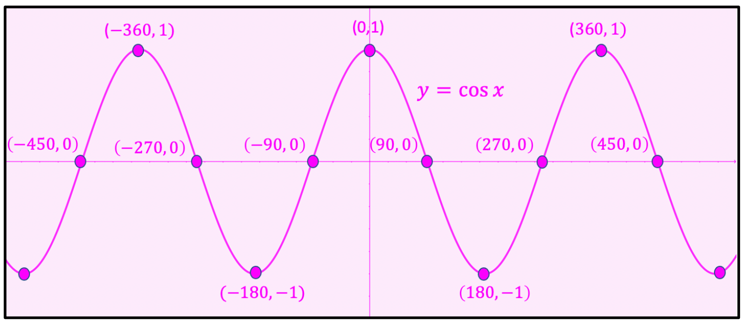
$\sin x$	$\cos x$	$\tan x$
$\sin 0° = 0 \Longrightarrow (0,0)$ $\sin 90° = 1 \Longrightarrow (90,0)$ $\sin 180° = 0 \Longrightarrow (180,0)$ $\sin 270° = -1 \Longrightarrow (270,-1)$ $\sin 360° = 0 \Longrightarrow (360,0)$ Once you have the positive x values the shape should be easy to spot for the negative x values etc (or you can use the diagrams clockwise to find the negative values too). The sin graph is a symmetric about the origin.	$\cos 0° = 1 \Longrightarrow (0,1)$ $\cos 90° = 0 \Longrightarrow (90,0)$ $\cos 180° = -1 \Longrightarrow (180,-1)$ $\cos 270° = 0 \Longrightarrow (270,0)$ $\cos 360° = 1 \Longrightarrow (360,0)$ Once you have the positive x values the shape should be easy to spot for the negative x values etc (or you can use the diagrams clockwise to find all the negative values too). The cos graph is symmetric about the y axis (y axis is a mirror line)	$\tan 0° = 0 \Longrightarrow (0,0)$ $\tan 90°$ =undef $\Longrightarrow$ asymptote $\tan 180° = 0 \Longrightarrow (180,0)$ $\tan 270°$ =undef $\Longrightarrow$ asymptote $\tan 360° = 0 \Longrightarrow (360,0)$ Once you have the positive x values the shape should be easy to spot for the negative x values etc (or you can use the diagrams clockwise to find all the negative values too).

The coordinates from the values have been indicated on the graphs below with a ●.

$y = \sin x$



$y = \cos x$



$y = \tan x$